Crypto Engineering 2025–2026

Symmetric encryption

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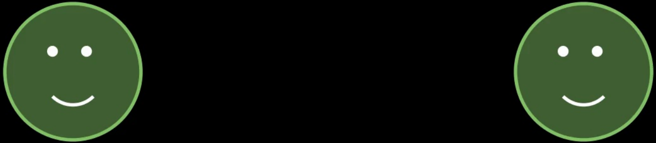
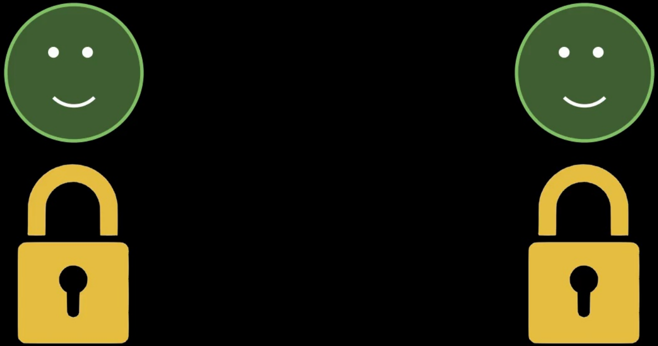
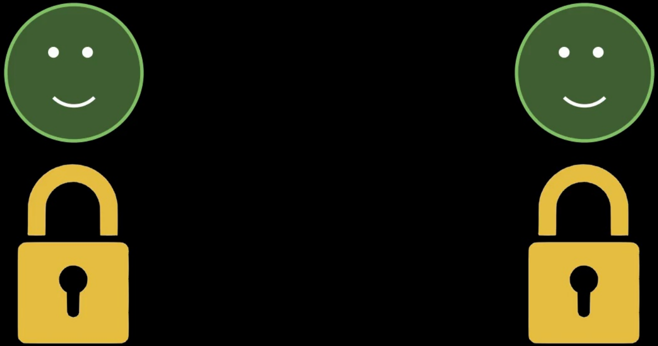
`https://leo.colisson.me/teaching/`

Reminder symmetric encryption & IND-CPA security





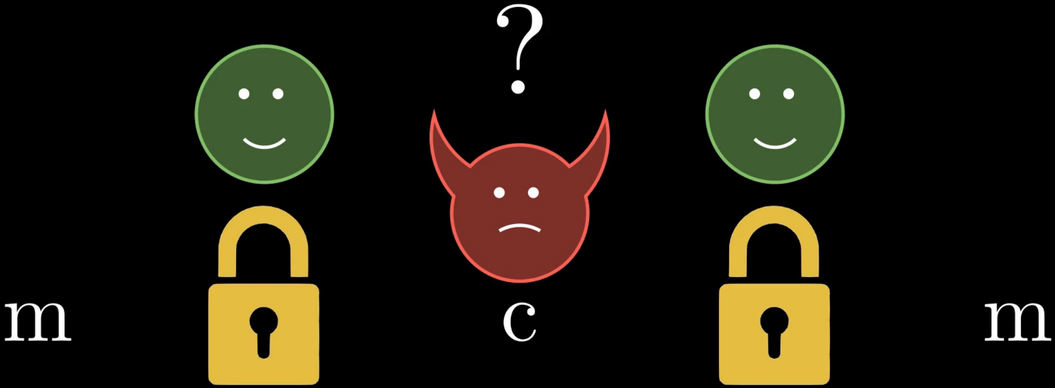
m



m

c


m
?
c


m
?
c
m

Symmetric encryption

Definition (Symmetric encryption scheme)

Let $\mathcal{K}$, $\mathcal{M}$, $\mathcal{C}$ be the set of, respectively, keys, messages and ciphertexts. An encryption scheme is a tuple $(\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ of polynomial algorithm:

Key $k \in \mathcal{K}$

Security parameter $\lambda \in \mathbb{N}$ in unary form:
Gen runs in poly time in the size of its input

- Key-generation $k \leftarrow \mathsf{Gen}(1^\lambda)$
- Encryption $c \leftarrow \mathsf{Enc}_k(m)$ ← Message $m \in \mathcal{M}$, sometimes written $\mathsf{Enc}(k, m)$.
- Decryption $m \leftarrow \mathsf{Dec}_k(c)$ ← Ciphertext $c \in \mathcal{C}$

that must be correct, i.e. such that for any $m \in \mathcal{M}$:

$$\Pr_{k \leftarrow \mathcal{K}} [\mathsf{Dec}_k(\mathsf{Enc}_k(m)) = m] = 1$$

One-Time Pad

Definition (One-Time Pad, OTP)

The One-Time Pad is the crypto-system defined as $\mathcal{M} = \mathcal{K} = \mathcal{C} = \{0,1\}^\lambda$ and $(\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ as:

```
OTP
Gen(1^λ):
  $k \xleftarrow{\$} \{0, 1\}^\lambda$
  return $k$
Enc(k, m):
  return $k \oplus m$
Dec(k, c):
  return $k \oplus c$
```

Correctness: $\forall k, \mathsf{Dec}(k, \mathsf{Enc}(k, m)) = k \oplus k \oplus m = m$. $\square$

Security of OTP

Last episode: we mentionned that the good notion of security is to distinguish between the encryption of one of two messages m_0 and m_1 chosen by the adversary. Still an important question:

What do we give to the adversary before they get to choose m_0 and m_1?

Can we reuse the key ?

Security of OTP

First (weak) security definition:

- We give NOTHING
- We change the key at any new encryption

More formally:

Definition (One-time secrecy)

An encryption scheme $\Sigma = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ with key-space $\mathcal{K}$, message-space $\mathcal{M}$ and cipher-text space $\mathcal{C}$ is *one-time secure* if:

```
$\mathcal{L}^{\Sigma}_{\text{ots-L}}$
EAVESDROP($m_L, m_R \in \mathcal{M}$):
    $k \leftarrow \mathsf{Gen}(1^\lambda)$
    return $\mathsf{Enc}_k(m_L)$
```

$$\equiv$$

```
$\mathcal{L}^{\Sigma}_{\text{ots-R}}$
EAVESDROP($m_L, m_R \in \mathcal{M}$):
    $k \leftarrow \mathsf{Gen}(1^\lambda)$
    return $\mathsf{Enc}_k(m_R)$
```

Security of OTP

Theorem

OTP is one-time secure

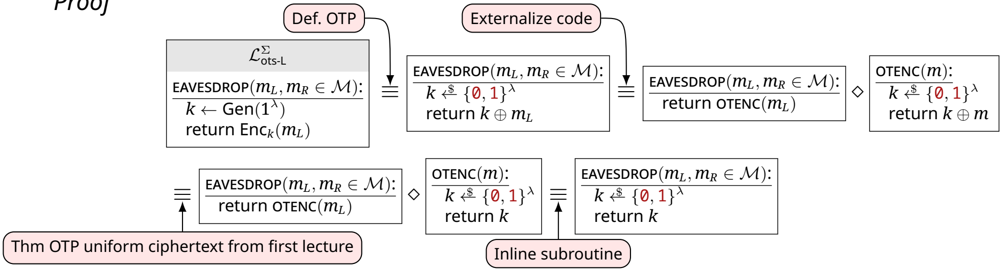
Proof (Exercice. See also the caseine exercice “OTP is one-time secure”)

Security of OTP

Theorem

OTP is one-time secure

Proof



Def. OTP

Externalize code

```
$\mathcal{L}^{\Sigma}_{\text{ots-L}}$
EAVESDROP$(m_L, m_R \in \mathcal{M})$:
    $k \leftarrow \mathsf{Gen}(1^\lambda)$
    return $\mathsf{Enc}_k(m_L)$
```

$\equiv$

```
EAVESDROP$(m_L, m_R \in \mathcal{M})$:
    $k \xleftarrow{\$} \{0,1\}^\lambda$
    return $k \oplus m_L$
```

$\equiv$

```
EAVESDROP$(m_L, m_R \in \mathcal{M})$:
    return OTENC$(m_L)$
```

$\diamond$

```
OTENC$(m)$:
    $k \xleftarrow{\$} \{0,1\}^\lambda$
    return $k \oplus m$
```

$\equiv$

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$\diamond$

```
OTENC$(m)$:
    $k \xleftarrow{\$} \{0,1\}^\lambda$
    return $k$
```

$\equiv$

```
EAVESDROP$(m_L, m_R \in \mathcal{M})$:
    $k \xleftarrow{\$} \{0,1\}^\lambda$
    return $k$
```

Thm OTP uniform ciphertext from first lecture

Inline subroutine

We realize that the last library does not depend on m_R or m_L at all. So we can apply all operations backward, except that we replace m_L with m_R to recover $\mathcal{L}^{\Sigma}_{\text{ots-R}} \equiv \mathcal{L}^{\Sigma}_{\text{ots-L}}$. □

Security OTP

Problem: Here, a **new key** k is re-sampled on every new encryption… **Highly impractical**! We would prefer to **re-use** the same key:

Definition (IND-CPA)

An encryption scheme $\Sigma = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ has indistinguishable security against *chosen-plaintext attacks* (IND-CPA security) if:

```
$\mathcal{L}^{\Sigma}_{\text{cpa-L}}$
$k \leftarrow \mathsf{Gen}(1^\lambda)$
EAVESDROP$(m_L, m_R \in \mathcal{M})$:
    return $\mathsf{Enc}_k(m_L)$
```

$\approx$

```
$\mathcal{L}^{\Sigma}_{\text{cpa-R}}$
$k \leftarrow \mathsf{Gen}(1^\lambda)$
EAVESDROP$(m_L, m_R \in \mathcal{M})$:
    return $\mathsf{Enc}_k(m_R)$
```

Security of OTP

?

Do you think that OTP is CPA secure? If yes, sketch a proof, if not, sketch an adversary and compute its advantage (see caseine first quiz question 9).

Ⓐ Yes

Ⓑ No

Security of OTP

Do you think that OTP is CPA secure? If yes, sketch a proof, if not, sketch an adversary and compute its advantage (see caseine first quiz question 9).

Ⓐ Yes ✗

Ⓑ No ✓ Exploit the fact that it is **deterministic encryption**:

Define

```
$\mathcal{A}$
$x \leftarrow$ EAVESDROP($0^\lambda$, $0^\lambda$)
$y \leftarrow$ EAVESDROP($0^\lambda$, $1^\lambda$)
return $x = y$
```

. Then, after inlining, we have

$\mathcal{A} \diamond \mathcal{L}^{\Sigma}_{\text{cpa-L}} =$

```
$\mathcal{A} \diamond \mathcal{L}^{\Sigma}_{\text{cpa-L}}$
$k \leftarrow$ Gen($1^\lambda$)
$x \leftarrow 0^\lambda \oplus k$
$y \leftarrow 0^\lambda \oplus k$
return $x = y$
```

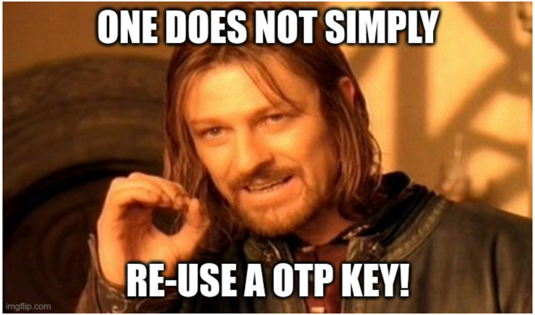
i.e. $\Pr\left[\mathcal{A} \diamond \mathcal{L}^{\Sigma}_{\text{cpa-L}} = 1\right] = 1$. But

$\mathcal{A} \diamond \mathcal{L}^{\Sigma}_{\text{cpa-L}} =$

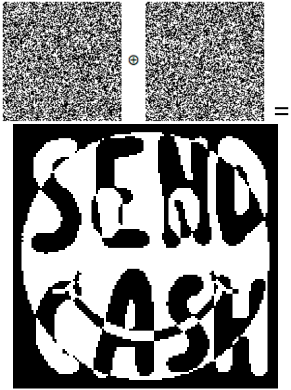
```
$\mathcal{A} \diamond \mathcal{L}^{\Sigma}_{\text{cpa-R}}$
$k \leftarrow$ Gen($1^\lambda$)
$x \leftarrow 0^\lambda \oplus k$
$y \leftarrow 1^\lambda \oplus k$
return $x = y$
```

i.e. $\Pr\left[\mathcal{A} \diamond \mathcal{L}^{\Sigma}_{\text{cpa-L}} = 1\right] = 0$. Adv $= 1 - 0 = 1 \neq$ negl(λ)

Security of OTP

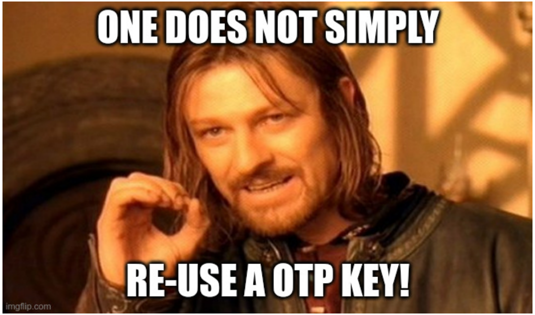


Never reuse a OTP key!!! This can lead to real attack:



More: `https://crypto.stackexchange.com/questions/59`, `https://incoherency.co.uk/blog/stories/otp-key-reuse.html`

Security of OTP



Never reuse a OTP key!!! This can lead to real attack:

More: `https://crypto.stackexchange.com/questions/59`, `https://incoherency.co.uk/blog/stories/otp-key-reuse.html`

How to build IND-CPA secure schemes

Encryption from simpler primitives

How to build an encryption:

- Approach 1: start from scratch. Less guarantees it will be secure.
- Approach 2 try to build encryption from simpler, more tested, primitives.

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Our approach

Encryption from simpler primitives

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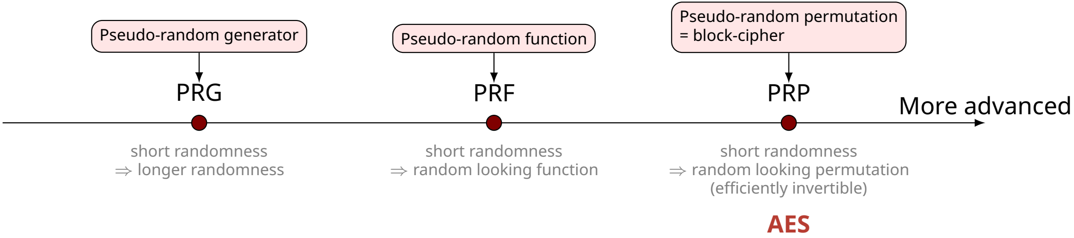
Our approach

But which more fundamental primitive can we use?



Pseudo-random generator

PRG

short randomness
⇒ longer randomness

Pseudo-random function

PRF

short randomness
⇒ random looking function

Pseudo-random permutation
= block-cipher

PRP

short randomness
⇒ random looking permutation
(efficiently invertible)

AES

More advanced

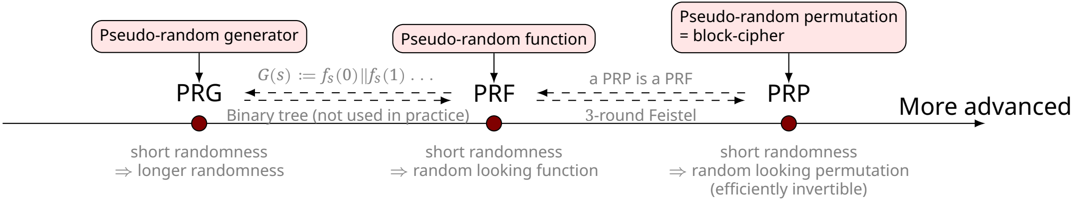
Encryption from simpler primitives

How to build an encryption:

- Approach 1: start from scratch. Less guarantees it will be secure.
- Approach 2 try to build encryption from simpler, more tested, primitives.

Our approach

But which more fundamental primitive can we use?



Pseudo-random generator

Pseudo-random function

Pseudo-random permutation
= block-cipher

PRG

$G(s) := f_s(0) \| f_s(1) \ldots$

Binary tree (not used in practice)

PRF

a PRP is a PRF

3-round Feistel

PRP

More advanced

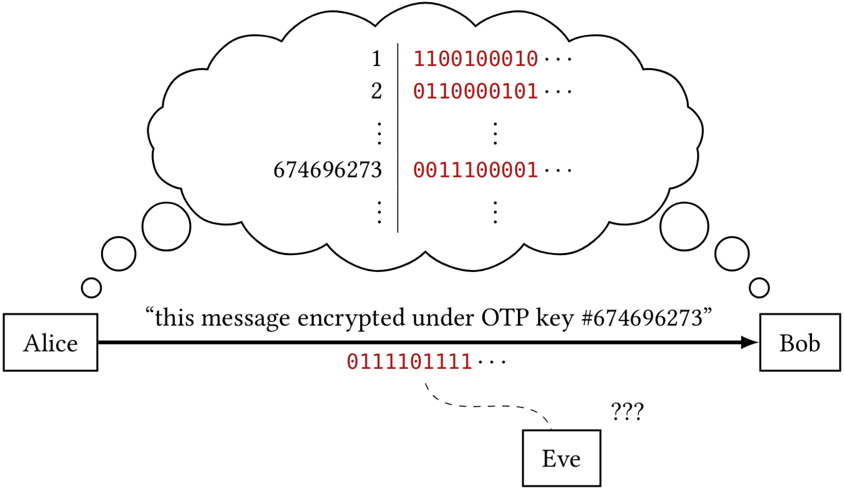
short randomness
$\Rightarrow$ longer randomness

short randomness
$\Rightarrow$ random looking function

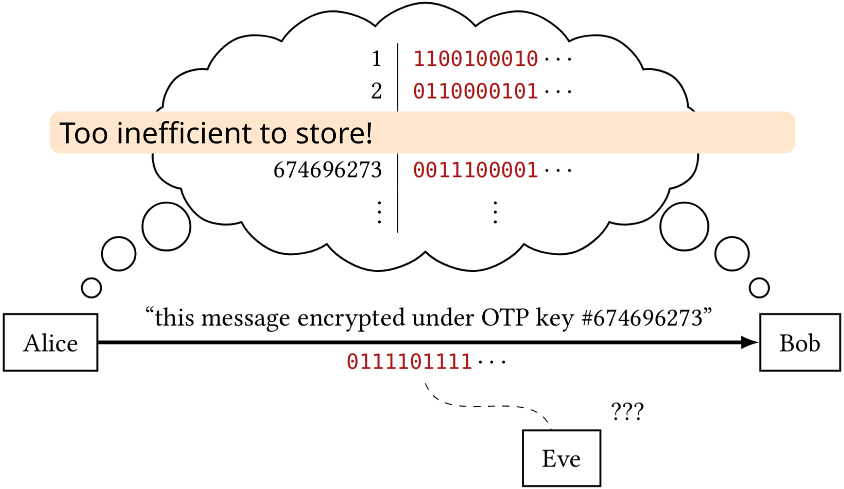
short randomness
$\Rightarrow$ random looking permutation
(efficiently invertible)

AES

Motivation PRF

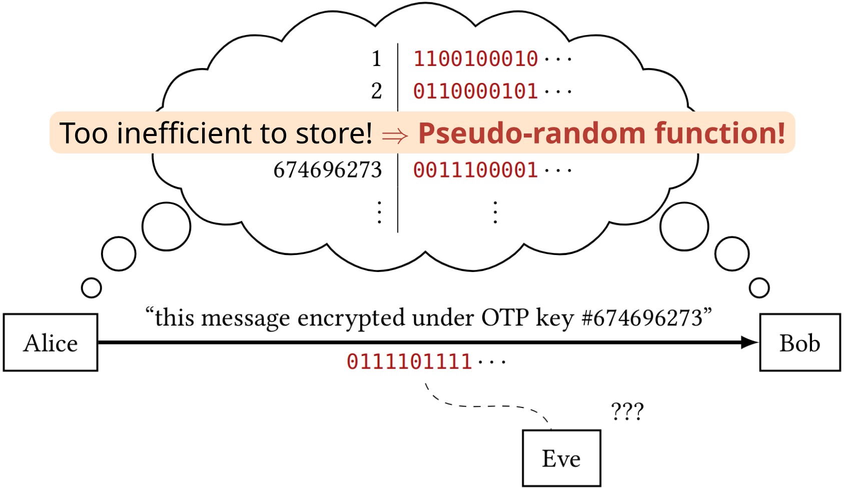


Motivation PRF



Too inefficient to store!

Motivation PRF



Too inefficient to store! ⇒ **Pseudo-random function!**

Pseudo-Random Generator (PRG)

PRG

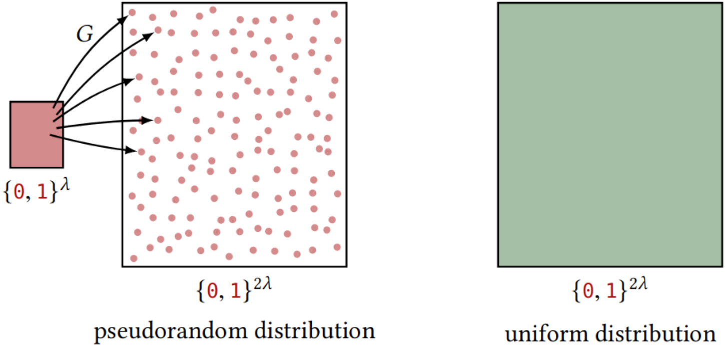
Let $G\colon \{0,1\}^\lambda \to \{0,1\}^{\lambda+l}$ be a deterministic function with $l > 0$. We say that G is a *secure Pseudo-Random Generator (PRG)* if:

```
$\mathcal{L}^{G}_{\text{prg-real}}$
QUERY():
    $s \xleftarrow{\$} \{0,1\}^\lambda$
    return $G(s)$
```

$$\approx$$

```
$\mathcal{L}^{G}_{\text{prg-ideal}}$
QUERY():
    $r \xleftarrow{\$} \{0,1\}^{\lambda+l}$
    return $r$
```

Pseudo-Random Generator (PRG)



PRG $\neq$ random number generator: small uniform source vs large non-uniform noise

Pseudo-Random Function (PRF)

PRF

Let $F \colon \{0,1\}^\lambda \times \{0,1\}^{\mathtt{in}} \to \{0,1\}^{\mathtt{out}}$ be a deterministic function. We say that F is a *secure Pseudo-Random Function (PRF)* if:

$\mathcal{L}^F_{\text{prf-real}}$

```
$k \xleftarrow{\$} \{0,1\}^\lambda$
LOOKUP($x \in \{0,1\}^{\mathtt{in}}$):
    return $F(k,x)$
```

$\approx$

$\mathcal{L}^F_{\text{prf-rand}}$

```
$T :=$ empty assoc. array
LOOKUP($x \in \{0,1\}^{\mathtt{in}}$):
    if $T[x]$ undefined:
        $T[x] \xleftarrow{\$} \{0,1\}^{\mathtt{out}}$
    return $T[x]$
```

Pseudo-Random Permutation (PRP)

PRP

Let $F\colon \{0,1\}^\lambda \times \{0,1\}^{\mathtt{blen}} \to \{0,1\}^{\mathtt{blen}}$ be a deterministic function. We say that F is a *secure Pseudo-Random Permutation (PRP)*, a.k.a. *block cipher*, if f is invertible, i.e. if there exists an efficient function F^{-1} such that $\forall x, k$:

$$F^{-1}(k, F(k,x)) = x$$

and if, after defining $T.\text{values} := \{v \mid \exists x, T[x] = v\}$, we have:

```
$\mathcal{L}^F_{\text{prp-real}}$
$k \xleftarrow{\$} \{0,1\}^\lambda$
LOOKUP($x \in \{0,1\}^{\mathtt{blen}}$):
    return $F(k,x)$
```

$$\approx$$

```
$\mathcal{L}^F_{\text{prp-rand}}$
$T :=$ empty assoc. array
LOOKUP($x \in \{0,1\}^{\mathtt{blen}}$):
    if $T[x]$ undefined:
        $T[x] \xleftarrow{\$} \{0,1\}^{\mathtt{blen}} \setminus T.\text{values}$
    return $T[x]$
```

PRP vs PRF

How far are PRP from PRF?

Natural attack: call $\textsc{Lookup}(x)$ on random x many times (say N) until we **find a collision** ($\textsc{Lookup}(x) = \textsc{Lookup}(x')$ for $x' \neq x$). If we can't find any, claim PRP, otherwise PRF.
Naively, think this has advantage $\approx \frac{1}{N}$, but much more efficient: $\approx \frac{1}{\sqrt{N}}$.

The birthday paradox

Birthday paradox = What is the probability of finding two persons with the same birthday in a class of 23 students?

- Ⓐ 7%
- Ⓑ 20%
- Ⓒ 50%

The birthday paradox

? **Birthday paradox** = What is the probability of finding two persons with the same birthday in a class of 23 students?

Ⓐ 7%

Ⓑ 20%

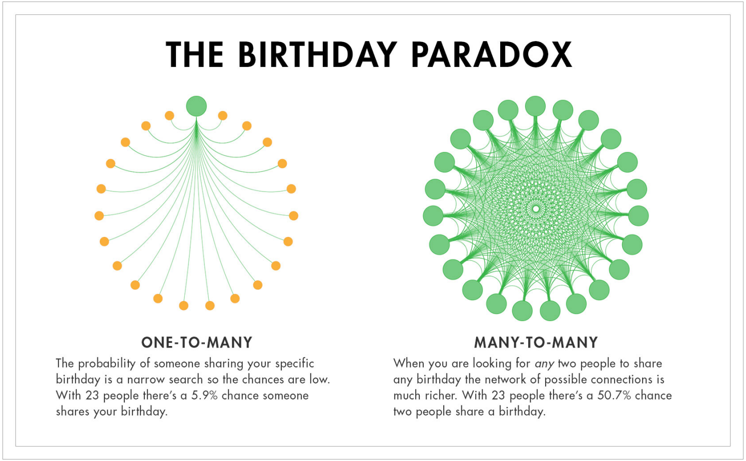
Ⓒ 50% ✓

If N = number of elements, n = number of samples, p = proba collision:

$$p(n) = 1 - \frac{N!}{(N-n)!}\frac{1}{N^n}$$

number of samples for proba collision $1/2 \approx \sqrt{N}$

The birthday paradox



THE BIRTHDAY PARADOX

ONE-TO-MANY

The probability of someone sharing your specific birthday is a narrow search so the chances are low. With 23 people there's a 5.9% chance someone shares your birthday.

MANY-TO-MANY

When you are looking for *any* two people to share any birthday the network of possible connections is much richer. With 23 people there's a 50.7% chance two people share a birthday.

`https://oddathenaeum.com/the-birthday-paradox/`

The birthday paradox

?

We said 2^{128} is HUGE. Is it doable to find a collision on a PRF/hash function with output size 128 bits?

- Ⓐ Yes, with a laptop
- Ⓑ Yes, with a GPU/ASIC cluster
- Ⓒ No

The birthday paradox

?

We said 2^{128} is HUGE. Is it doable to find a collision on a PRF/hash function with output size 128 bits?

Ⓐ Yes, with a laptop

Ⓑ Yes, with a GPU/ASIC cluster ✓ $\sqrt{2^{128}} = 2^{128/2} = 2^{64}$.
$\Rightarrow$ First course, 2^{64} doable with GPU/ASIC cluster.

Ⓒ No

The birthday paradox

But asymptotically, the birthday paradox does not cause issues:

Theorem (Asymptotic birthday paradox)

We have

```
$\mathcal{L}_{\text{samp-L}}$
SAMP ():
    $r \xleftarrow{\$} \{0,1\}^\lambda$
    return $r$
```

$\approx$

```
$\mathcal{L}_{\text{samp-R}}$
$R := \emptyset$
SAMP ():
    $r \xleftarrow{\$} \{0,1\}^\lambda \setminus R$
    $R = R \cup \{r\}$
    return $r$
```

.

Proof. See Caseine exercise “Asymptotic birthday paradox”.

The birthday paradox

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.

```
$\mathcal{L}_{\text{samp-L}}$
SAMP():
    $r \xleftarrow{\$} \{0,1\}^\lambda$
    return $r$
```

$=$

```
$\mathcal{L}_{\text{samp-R}}$
$R := \emptyset$
bad $:= 0$
SAMP():
    $r \xleftarrow{\$} \{0,1\}^\lambda$
    if $r \in R$:
        bad $:= 1$

    $R = R \cup \{r\}$
    return $r$
```

$\approx$

```
$\mathcal{L}_{\text{samp-R}}$
$R := \emptyset$
bad $:= 0$
SAMP():
    $r \xleftarrow{\$} \{0,1\}^\lambda$
    if $r \in R$:
        bad $:= 1$
        $r \xleftarrow{\$} \{0,1\}^\lambda \setminus R$
    $R = R \cup \{r\}$
    return $r$
```

$=$

```
$\mathcal{L}_{\text{samp-R}}$
$R := \emptyset$
SAMP():
    $r \xleftarrow{\$} \{0,1\}^\lambda \setminus R$
    $R = R \cup \{r\}$
    return $r$
```

Proof. Bad-event lemma: $\mathcal{A}$ is polynomial, so

$$\Pr[\,\text{bad} = 1\,] = \text{poly}(\lambda) \times \frac{\text{poly}(\lambda)}{2^\lambda} = \text{negl}(\lambda) \qquad \square$$

$= |R|$

Number of calls to SAMP

The birthday paradox

Take-home message

The birthday paradox does not harm asymptotic security ($\sqrt{\mathsf{negl}(\lambda)} = \mathsf{negl}(\lambda)$), but in real life, the **size of the key may need to be doubled** to prevent this attack.

A PRP is a PRF

A PRP is a PRF

Let $F\colon \{\texttt{0},\texttt{1}\}^\lambda \times \{\texttt{0},\texttt{1}\}^\lambda \to \{\texttt{0},\texttt{1}\}^\lambda$ be a secure PRP (with blen $= \lambda$). Then F is also a secure PRF.

Proof. Caseine exercise “A PRP is a PRF”

A PRP is a PRF

A PRP is a PRF

Let $F\colon \{0,1\}^\lambda \times \{0,1\}^\lambda \to \{0,1\}^\lambda$ be a secure PRP (with $\text{blen} = \lambda$). Then F is also a secure PRF.

Proof.

$$\mathcal{L}_{\text{prf-real}} \overset{\text{Identical}}{\equiv} \mathcal{L}_{\text{prp-real}} \overset{\text{Def PRP}}{\approx}$$

$\mathcal{L}^F_{\text{prp-rand}}$

```
$T :=$ empty assoc. array
LOOKUP($x \in \{0,1\}^{\text{blen}}$):
  if $T[x]$ undefined:
    $T[x] \xleftarrow{\$} \{0,1\}^{\text{blen}} \setminus T.\text{values}$
  return $T[x]$
```

$$\overset{\text{Externalize}}{\equiv}$$

$\mathcal{L}^F_1$

```
$T :=$ empty assoc. array
LOOKUP($x \in \{0,1\}^{\text{blen}}$):
  if $T[x]$ undefined:
    $T[x] \xleftarrow{\$}$ SAMP()
  return $T[x]$
```

$$\diamond$$

$\mathcal{L}_{\text{samp-R}}$

```
$R := \emptyset$
SAMP ():
  $r \xleftarrow{\$} \{0,1\}^\lambda \setminus R$
  $R = R \cup \{r\}$
  return $r$
```

$$\overset{\text{Birthday paradox}}{\approx}$$

$\mathcal{L}^F_1$

```
$T :=$ empty assoc. array
LOOKUP($x \in \{0,1\}^{\text{blen}}$):
  if $T[x]$ undefined:
    $T[x] \xleftarrow{\$}$ SAMP()
  return $T[x]$
```

$$\diamond$$

$\mathcal{L}_{\text{samp-L}}$

```
SAMP ():
  $r \xleftarrow{\$} \{0,1\}^\lambda$
  return $r$
```

$$\underset{\text{Inline}}{\equiv}$$

$\mathcal{L}^F_{\text{prf-rand}}$

```
$T :=$ empty assoc. array
LOOKUP($x \in \{0,1\}^{\text{in}}$):
  if $T[x]$ undefined:
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  return $T[x]$
```

□

How to build IND-CPA schemes from PRF or block-ciphers?

IND-CPA from PRF

Based on above idea, first (not so efficient) solution:

Definition (PRF pseudo-OTP)

Let F be a secure PRF. We define the PRF pseudo-OTP encryption scheme as $\mathcal{K} = \{0,1\}^\lambda$, $\mathcal{M} = \{0,1\}^{\text{out}}$, $\mathcal{C} = \{0,1\}^\lambda \times \{0,1\}^{\text{out}}$, and:

```
$\Sigma_{\text{prf-pseudo-OTP}}$
Gen() :
  $k \xleftarrow{\$} \{0,1\}^\lambda$
  return $k$
Enc($k, m$) :
  $r \xleftarrow{\$} \{0,1\}^\lambda$
  $x := F(k,r) \oplus m$
  return $(r, x)$
Dec($k, c$) :
  $m := F(k,r) \oplus c$
  return $m$
```

IND-CPA from PRF

Theorem (security PRF pseudo-OTP)

The PRF pseudo-OTP is IND-CPA secure.

? Exercice: try to prove its security (answer next slide)

$\mathcal{L}^{\Sigma}_{\text{cpa-L}}$

```
$k \leftarrow \mathsf{Gen}(1^\lambda)$
$\textsc{eavesdrop}(m_L, m_R \in \mathcal{M})$:
    return $\mathsf{Enc}_k(m_L)$
```

Def. Enc

$\equiv$

$\mathcal{L}_1$

```
$k \leftarrow \mathsf{Gen}(1^\lambda)$
$\textsc{eavesdrop}(m_L, m_R \in \mathcal{M})$:
    $r \xleftarrow{\$} \{0,1\}^\lambda$
    $x := F(k, r) \oplus m_L$
    return $(r, x)$
```

Externalize

$\equiv$

$\mathcal{L}_1$

```
$\textsc{eavesdrop}(m_L, m_R \in \mathcal{M})$:
    $r \xleftarrow{\$} \{0,1\}^\lambda$
    $x := \textsc{lookup}(r) \oplus m_L$
    return $(r, x)$
```

$\diamond$

$\mathcal{L}^{F}_{\text{prf-real}}$

```
$k \xleftarrow{\$} \{0,1\}^\lambda$
$\textsc{lookup}(x \in \{0,1\}^{\mathtt{in}})$:
    return $F(k, x)$
```

$\mathcal{L}^{\Sigma}_{\text{cpa-L}}$

```
$k \leftarrow \mathsf{Gen}(1^\lambda)$
$\textsc{eavesdrop}(m_L, m_R \in \mathcal{M})$:
    return $\mathsf{Enc}_k(m_L)$
```

Def. Enc $\equiv$

$\mathcal{L}_1$

```
$k \leftarrow \mathsf{Gen}(1^\lambda)$
$\textsc{eavesdrop}(m_L, m_R \in \mathcal{M})$:
    $r \overset{\$}{\leftarrow} \{0,1\}^\lambda$
    $x := F(k, r) \oplus m_L$
    return $(r, x)$
```

Externalize $\equiv$

$\mathcal{L}_1$

```
$\textsc{eavesdrop}(m_L, m_R \in \mathcal{M})$:
    $r \overset{\$}{\leftarrow} \{0,1\}^\lambda$
    $x := \textsc{lookup}(r) \oplus m_L$
    return $(r, x)$
```

$\diamond$

$\mathcal{L}^{F}_{\text{prf-real}}$

```
$k \overset{\$}{\leftarrow} \{0,1\}^\lambda$
$\textsc{lookup}(x \in \{0,1\}^{\texttt{in}})$:
    return $F(k, x)$
```

$\mathcal{L}_1$

$\mathcal{L}^{F}_{\text{prf-rand}}$

```
$\mathcal{L}_1$
$k \leftarrow \mathsf{Gen}(1^\lambda)$
EAVESDROP($m_L, m_R \in \mathcal{M}$):
    $r \xleftarrow{\$} \{0,1\}^\lambda$
    $x := F(k, r) \oplus m_L$
    return $(r, x)$
```

Def. Enc

$\equiv$

Externalize

```
$\mathcal{L}_1$
EAVESDROP($m_L, m_R \in \mathcal{M}$):
    $r \xleftarrow{\$} \{0,1\}^\lambda$
    $x :=$ LOOKUP($r$) $\oplus m_L$
    return $(r, x)$
```

$\diamond$

```
$\mathcal{L}^F_{\text{prf-real}}$
$k \xleftarrow{\$} \{0,1\}^\lambda$
LOOKUP($x \in \{0,1\}^{\texttt{in}}$):
    return $F(k, x)$
```

$\equiv$

Def PRF

```
$\mathcal{L}_1$
EAVESDROP($m_L, m_R \in \mathcal{M}$):
    $r \xleftarrow{\$} \{0,1\}^\lambda$
    $x :=$ LOOKUP($r$) $\oplus m_L$
    return $(r, x)$
```

$\diamond$

```
$\mathcal{L}^F_{\text{prf-rand}}$
$T :=$ empty assoc. array
LOOKUP($x \in \{0,1\}^{\texttt{in}}$):
    if $T[x]$ undefined:
        $T[x] \xleftarrow{\$} \{0,1\}^{\texttt{out}}$
    return $T[x]$
```

$\approx$

$\mathcal{L}$

Externalize $\equiv$

```
EAVESDROP($m_L, m_R \in \mathcal{M}$):
    $r \xleftarrow{\$} \{0,1\}^\lambda$
    $x :=$ LOOKUP($r$) $\oplus m_L$
    return $(r, x)$
```

$\diamond$

```
$k \xleftarrow{\$} \{0,1\}^\lambda$
LOOKUP($x \in \{0,1\}^{\texttt{in}}$):
    return $F(k, x)$
```

Def PRF $\approx$

$\mathcal{L}_1$

```
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    $r \xleftarrow{\$} \{0,1\}^\lambda$
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```

$\diamond$

$\mathcal{L}^F_{\text{prf-rand}}$

```
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LOOKUP($x \in \{0,1\}^{\texttt{in}}$):
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    return $T[x]$
```

Inline $\equiv$

$\mathcal{L}_2$

```
$T :=$ empty assoc. array
EAVESDROP($m_L, m_R \in \mathcal{M}$):
    $r \xleftarrow{\$} \{0,1\}^\lambda$
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        $T[r] \xleftarrow{\$} \{0,1\}^{\texttt{out}}$
    $x := T[r] \oplus m_L$
    return $(r, x)$
```

```
    return $(r, x)$
```

```
    $T[x] \xleftarrow{\$} \{0,1\}^{\mathsf{out}}$
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```
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  $x := T[r] \oplus m_L$
  return $(r, x)$
```

Externalize $\equiv$

```
$\mathcal{L}_2$
$T :=$ empty assoc. array
EAVESDROP$(m_L, m_R \in \mathcal{M})$:
  $r \xleftarrow{\$}$ SAMP()
  if $T[r]$ undefined:
    $T[r] \xleftarrow{\$} \{0,1\}^{\mathsf{out}}$
  $x := T[r] \oplus m_L$
  return $(r, x)$
```

$\diamond$

```
$\mathcal{L}_{\text{samp-L}}$
SAMP ():
  $r \xleftarrow{\$} \{0,1\}^{\lambda}$
  return $r$
```

Externalize $\equiv$

$\mathcal{L}_2$

```
$T :=$ empty assoc. array
EAVESDROP($m_L, m_R \in \mathcal{M}$):
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    if $T[r]$ undefined:
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    return $(r, x)$
```

$\diamond$

$\mathcal{L}_{\text{samp-L}}$

```
SAMP():
    $r \xleftarrow{\$} \{0,1\}^\lambda$
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```

Asymptotic birthday paradox $\approx$

$\mathcal{L}_2$

```
$T :=$ empty assoc. array
EAVESDROP($m_L, m_R \in \mathcal{M}$):
    $r \xleftarrow{\$}$ SAMP()
    if $T[r]$ undefined:
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    $x := T[r] \oplus m_L$
    return $(r, x)$
```

$\diamond$

$\mathcal{L}_{\text{samp-R}}$

```
$R := \emptyset$
SAMP():
    $r \xleftarrow{\$} \{0,1\}^\lambda \setminus R$
    $R = R \cup \{r\}$
    return $r$
```

$\mathcal{L}_3$

$\mathcal{L}_2$

Asymptotic birthday paradox $\approx$

```
$T :=$ empty assoc. array
EAVESDROP($m_L, m_R \in \mathcal{M}$):
    $r \xleftarrow{\$}$ SAMP()
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```

$\diamond$

$\mathcal{L}_{\text{samp-R}}$

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Inline $\equiv$

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    if $T[r]$ undefined:
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    $x := T[r] \oplus m_L$
    return $(r, x)$
```

$\mathcal{L}_4$

Inline $\equiv$

$\mathcal{L}_3$

```
$T :=$ empty assoc. array
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EAVESDROP($m_L, m_R \in \mathcal{M}$):
    $r \xleftarrow{\$} \{0,1\}^{\lambda} \setminus R$
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    if $T[r]$ undefined:
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    return $(r, x)$
```

$T[r]$ always undefined $\equiv$

$\mathcal{L}_4$

```
EAVESDROP($m_L, m_R \in \mathcal{M}$):
    $r \xleftarrow{\$} \{0,1\}^{\lambda} \setminus R$
    $R = R \cup \{r\}$
    $r' \xleftarrow{\$} \{0,1\}^{\mathtt{out}}$
    $x := r' \oplus m_L$
    return $(r, x)$
```

$\mathcal{L}_5$

```
    if $T[r]$ undefined:
        $T[r] \xleftarrow{\$} \{0,1\}^{\text{out}}$
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    return $(r, x)$
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$\mathcal{L}_4$

```
EAVESDROP($m_L, m_R \in \mathcal{M}$):
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```

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```
EAVESDROP($m_L, m_R \in \mathcal{M}$):
    $r \xleftarrow{\$} \{0,1\}^{\lambda} \setminus R$
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    return $(r, x)$
```

Externalize $\equiv$

$\diamond$

$\mathcal{L}_{\text{otp-real}}$

```
OTENC($m \in \{0,1\}^{\lambda}$):
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    return $k \oplus m$
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T[r] always undefined $\equiv$

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    return $(r, x)$
```

$\diamond$

$\mathcal{L}_{\text{otp-rand}}$

```
OTENC($m \in \{0,1\}^\lambda$):
    $c \xleftarrow{\$} \{0,1\}^\lambda$
    return $c$
```

OTP uniform ciphertext $\equiv$

```
    $x \leftarrow \text{OTENC}(m_L)$
    return $(r, x)$
```

```
    return $k \oplus m$
```

OTP uniform ciphertext $\equiv$

```
$\mathcal{L}_5$
EAVESDROP($m_L, m_R \in \mathcal{M}$):
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```

$\diamond$

```
$\mathcal{L}_{\text{otp-rand}}$
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```

Inline $\equiv$

```
$\mathcal{L}_5$
EAVESDROP($m_L, m_R \in \mathcal{M}$):
    $r \xleftarrow{\$} \{0,1\}^\lambda \setminus R$
    $R = R \cup \{r\}$
    $x \xleftarrow{\$} \{0,1\}^\lambda$
    return $(r, x)$
```

Since this last library is symmetric with respect to m_L and m_R, we can do exactly the same computations starting from $\mathcal{L}^{\Sigma}_{\text{cpa-R}}$ and we will find the exact same library (or, equivalently, do the operations backward with m_R instead of m_L), hence $\mathcal{L}^{\Sigma}_{\text{cpa-R}} \approx \mathcal{L}^{\Sigma}_{\text{cpa-L}}$.

Limitations PRF pseudo-OTP

Good to have secure IND-CPA scheme, but **how do we encrypt an arbitrary long message m**?

- First idea: **cut m in chunks** of length $\{0, 1\}^{\text{out}}$, and encrypt them separately.
  $\Rightarrow$ Issue: remember, Enc is a tupple (r, x), i.e. for l chunks, overhead of λl
  Too inefficient! 😰

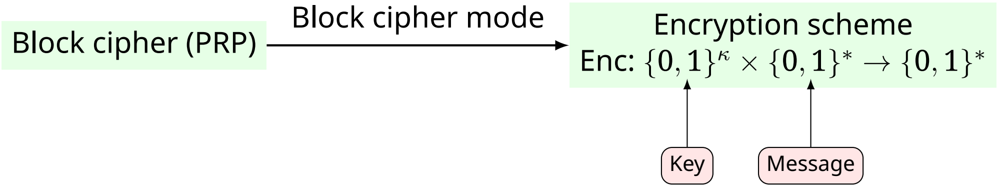
Limitations PRF pseudo-OTP

Good to have secure IND-CPA scheme, but **how do we encrypt an arbitrary long message m**?

- First idea: **cut m in chunks** of length $\{0, 1\}^{\text{out}}$, and encrypt them separately.
  $\Rightarrow$ Issue: remember, Enc is a tupple (r, x), i.e. for l chunks, overhead of λl
  Too inefficient! 😢
- Solution: use **block cipher modes**!

Block cipher modes

Multiple modes of operation (= variants):



Block cipher (PRP)

Block cipher mode

Encryption scheme

Enc: $\{0,1\}^\kappa \times \{0,1\}^* \to \{0,1\}^*$

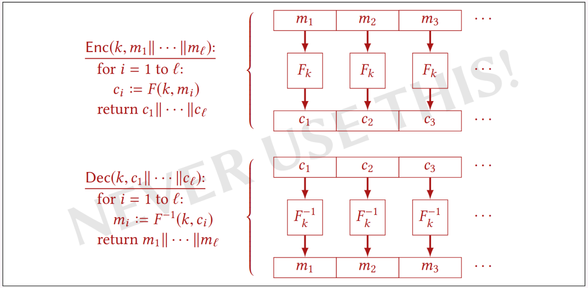
Key

Message

Common modes

Definition (ECB mode: NEVER USE THIS)

The (INSECURE!) Electronic Codebook (ECB) mode is defined as:



```
$\underline{\mathrm{Enc}(k, m_1 \| \cdots \| m_\ell)}$:
  for $i = 1$ to $\ell$:
    $c_i := F(k, m_i)$
  return $c_1 \| \cdots \| c_\ell$

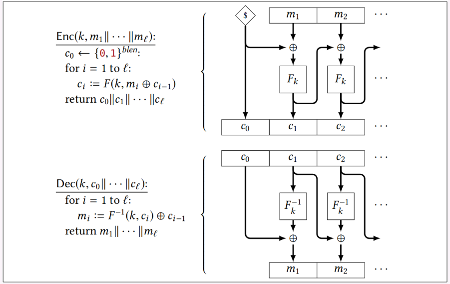
$\underline{\mathrm{Dec}(k, c_1 \| \cdots \| c_\ell)}$:
  for $i = 1$ to $\ell$:
    $m_i := F^{-1}(k, c_i)$
  return $m_1 \| \cdots \| m_\ell$
```

? This mode is said to be worse than deterministic. Find an attack that make a single call to the encryption function.

Common modes

Definition (CBC mode)

The Cipher Block Chaining (CBC) mode is defined as:



```
$\text{Enc}(k, m_1 \| \cdots \| m_\ell)$:
  $c_0 \leftarrow \{0,1\}^{blen}$;
  for $i = 1$ to $\ell$:
    $c_i := F(k, m_i \oplus c_{i-1})$
  return $c_0 \| c_1 \| \cdots \| c_\ell$

$\text{Dec}(k, c_0 \| \cdots \| c_\ell)$:
  for $i = 1$ to $\ell$:
    $m_i := F^{-1}(k, c_i) \oplus c_{i-1}$
  return $m_1 \| \cdots \| m_\ell$
```

c_0 is called the initialization vector (IV). Why can't we set it to a fixed value?

- Ⓐ It acts like a OTP key on the message, hence hides it
- Ⓑ Used to have a non-deterministic encryption

Common modes

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The Cipher Block Chaining (CBC) mode is defined as:

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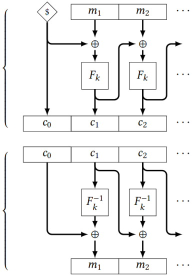
  return $c_0\|c_1\|\cdots\|c_\ell$
```



```
$\text{Dec}(k, c_0\|\cdots\|c_\ell)$:
  for $i = 1$ to $\ell$:
    $m_i := F^{-1}(k, c_i) \oplus c_{i-1}$
  return $m_1\|\cdots\|m_\ell$
```

?

c_0 is called the initialization vector (IV). Why can't we set it to a fixed value?

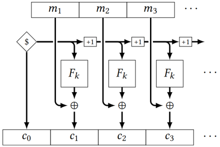
- Ⓐ It acts like a OTP key on the message, hence hides it ✗ IV is public, so cannot be a OTP key!
- Ⓑ Used to have a non-deterministic encryption ✓

Common modes

Definition (CTR mode)

The counter (CTR) mode is defined as:

```
$\mathsf{Enc}(k, m_1 \| \cdots \| m_\ell)$:
    $r \leftarrow \{0,1\}^{blen}$
    $c_0 := r$
    for $i = 1$ to $\ell$:
        $c_i := F(k, r) \oplus m_i$
        $r := r + 1 \% 2^{blen}$
    return $c_0 \| \cdots \| c_\ell$
```



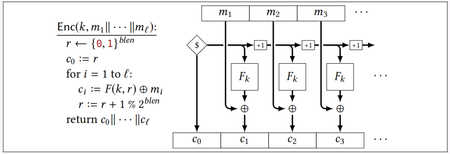
Try to find the decryption algorithm. Do you need to compute F^{-1}?

- Ⓐ Yes
- Ⓑ No

Common modes

Definition (CTR mode)

The counter (CTR) mode is defined as:



```
$\mathsf{Enc}(k, m_1 \| \cdots \| m_\ell)$:
    $r \leftarrow \{0, 1\}^{blen}$
    $c_0 := r$
    for $i = 1$ to $\ell$:
        $c_i := F(k, r) \oplus m_i$
        $r := r + 1 \% 2^{blen}$
    return $c_0 \| \cdots \| c_\ell$
```

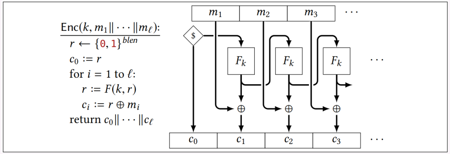
Try to find the decryption algorithm. Do you need to compute F^{-1}?

- Ⓐ Yes ✗
- Ⓑ No ✓ No need to have a PRP, PRF is enough (but in practice, most efficient PRF are PRP anyway)

Common modes

Definition (OFB mode)

The output feedback (OFB) mode is defined as:



```
$\text{Enc}(k, m_1 \| \cdots \| m_\ell)$:
    $r \leftarrow \{0, 1\}^{blen}$
    $c_0 := r$
    for $i = 1$ to $\ell$:
        $r := F(k, r)$
        $c_i := r \oplus m_i$
    return $c_0 \| \cdots \| c_\ell$
```

?

Try to find the decryption algorithm. Do you need to compute F^{-1}?

Ⓐ Yes

Ⓑ No

Common modes

Definition (OFB mode)

The output feedback (OFB) mode is defined as:

```
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        $r := F(k, r)$
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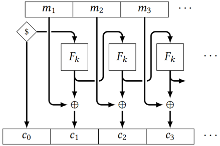
```



?

Try to find the decryption algorithm. Do you need to compute F^{-1}?

- Ⓐ Yes ✗
- Ⓑ No ✓ No need to have a PRP, PRF is enough

Comparison of modes

	ECB	CBC	CTR	OFB
IND-CPA	✗ !!!	✓	✓	✓
Parallelizable		✗	✓	✗
Pre-computable		✗	✓	✓
Can avoid padding		✗	✓	✓
Safer with no permutation cycle		✗	✓	✗
Slightly safer against IV re-use (e.g. in bad implementation)		✓	✗	✗

Winner is CTR mode! (but wait encrypt & authenticate modes like GCM)

Comparison of modes

?

A friend proposes to encode your hard drive with AES in OFB mode. Is this a good idea? Why?

Ⓐ Yes

Ⓑ No

Comparison of modes

?

A friend proposes to encode your hard drive with AES in OFB mode. Is this a good idea? Why?

Ⓐ Yes ✗

Ⓑ No ✓ Mauvaise idée : le mode OFB n’est pas parallélisable. Il faut donc déchiffrer tout le disque pour accéder au dernier octet !

Modes vulnerable to birthday attacks

All modes are vulnerable to birthday attacks (cf TD), so make sure you encrypt less than $2^{\text{blen}/2}$ blocks (i.e. keep blen large, e.g. don't use 3DES! (64 bits)).

Today: most widely used cipher is

Advanced Encryption Standard (AES)

with 128 bits block length (key length: 128, 192 or 256 bits). See also:

- Rijndael (generalization AES): block length 128, 192, or 256,
- Serpent (2nd finalist in Advanced Encryption Standard process)
- Twofish ($\text{blen} = 128$) and blowfish (warning: $\text{blen} = 64$!)
- never use DES = broken (previous standard), temporarily replaced by 3DES

IND-CPA for variable-length plaintexts

Can you find a generic IND-CPA attack against these cipher modes of operation (e.g. CTR, assume blen $= \lambda$ for simplicity)?

Ⓐ No

Ⓑ Yes, with

```
$\mathcal{A}$
$c :=$ EAVESDROP$(0^\lambda, 0^\lambda)$
$d :=$ EAVESDROP$(0^\lambda, 1^\lambda)$
return $c \stackrel{?}{=} d$
```

Ⓒ Yes, with

```
$\mathcal{A}$
$c :=$ EAVESDROP$(0^\lambda, 0^{2\lambda})$
return $|c| \stackrel{?}{=} 2\lambda$
```

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?

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```

✗

Ⓒ Yes, with

```
$\mathcal{A}$
$c :=$ EAVESDROP$(0^\lambda, 0^{2\lambda})$
return $|c| \overset{?}{=} 2\lambda$
```

✓ The length of the ciphertext equals $\lambda + |m| \Rightarrow$ **leaks the length** of the message!

IND-CPA for variable-length plaintexts

IND-CPA for variable-length plaintexts

When messages can have various length, we need to update the definition of security:

```
$\mathcal{L}^{\Sigma}_{\text{cpa-L}}$
$k \leftarrow \text{Gen}(1^\lambda)$
EAVESDROP$(m_L, m_R \in \mathcal{M})$:
    if $|m_L| \neq |m_R|$ return err
    return $\text{Enc}_k(m_L)$
```

$\approx$

```
$\mathcal{L}^{\Sigma}_{\text{cpa-R}}$
$k \leftarrow \text{Gen}(1^\lambda)$
EAVESDROP$(m_L, m_R \in \mathcal{M})$:
    if $|m_L| \neq |m_R|$ return err
    return $\text{Enc}_k(m_R)$
```

IND-CPA for variable-length plaintexts

Is leaking the length an issue?

Sometimes! E.g.:

- Google maps sends tiles, each tile having a different size (despite same pixel size) due to compression $\Rightarrow$ possible to know what tile is displayed only by looking at traffic
- Variable-bit-rate (VBR) in video shows different (chunk of) “frame” size depending on the time. Possible to know which movie you watch on netflix/youtube based on this, and even identity speaker/language/word spoken in voice chat programs!

Padding

Padding

What if $|m|$ is not a multiple of the block length?

- CTR mode: simple, just truncate the ciphertext (like regular OTP)
- CBC mode: need to add **padding** (add data until reaching block length) (also possible to do “ciphertext stealing” in this specific case)

Padding

Many ways to pad m into m':

- add zeros: not working! When decrypting, how do you how many zeros to remove?
- ANSI X.923 standard: add `0`'s until the last byte that contains the number of padded bytes
- PKCS#7 standard: if b bytes of padding needed, add the actual b byte b times
- ISO/IEC 7816-4 standard: append `10`$\dots$`0`

The actual choice has **little importance**, not really a security feature (at least when considering passive adversaries, see later)

Padding

?

Consider ISO/IEC 7816-4 standard (append `10`...`0`): if you pad a message m of size $k$blen into m', what is the size of m'?

- Ⓐ $k$blen
- Ⓑ $k\text{blen} + 1$
- Ⓒ $(k+1)$blen

Padding

?

Consider ISO/IEC 7816-4 standard (append $\texttt{10}\ldots\texttt{0}$): if you pad a message m of size $k$blen into m', what is the size of m'?

Ⓐ $k$blen ✗

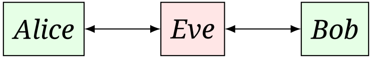
Ⓑ $k$blen $+ 1$ ✗

Ⓒ $(k+1)$blen ✓ (like all paddings, it increases the size of the message)

Padding oracle attack & CCA security

Padding oracle attack

Before: passive adversary (somewhat unrealistic). Now, we consider **active** adversaries:



What happens if Bob returns an error if the padding is incorrect?

Padding oracle attack

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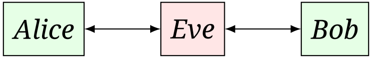


What happens if Bob returns an error if the padding is incorrect?
⇒ Eve can completely recover the encrypted message!

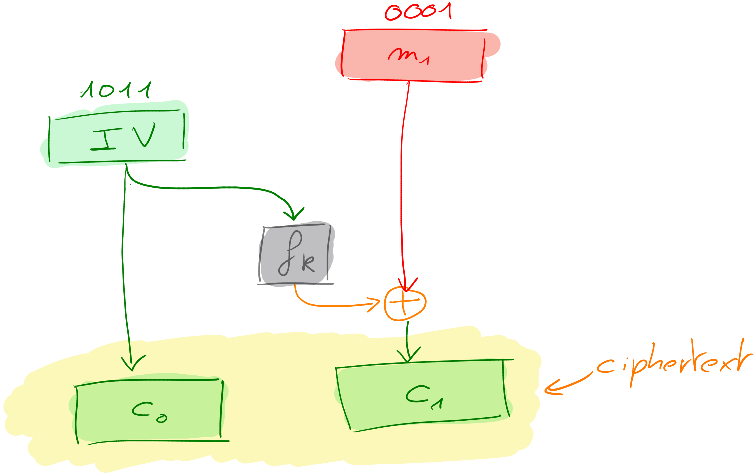
Padding oracle attack (illustrated on board)

Attack model: CTR mode, padding ANSI X.923, $\mathcal{A}$ has access to .

```
$k \leftarrow \mathsf{Gen}(1^\lambda)$
PADDINGORACLE($c$):
    $m := \mathsf{Dec}(k, c)$
    return VALIDPAD($m$)
```

(hence VALIDPAD(m) checks if m ends with a byte b containing before $b - 1$ bytes filled with `0`'s).
Say that we have access to $c_0 \leftarrow \mathsf{Enc}_k(m_0)$ (where m_0 is already padded), goal is to find m_0.

Padding oracle attack



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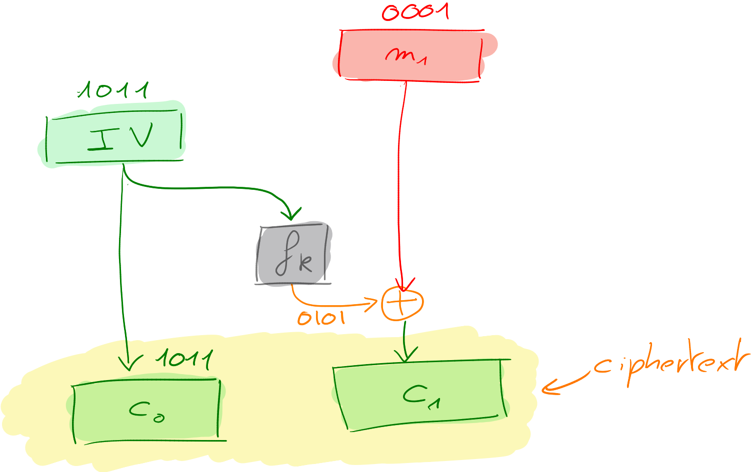

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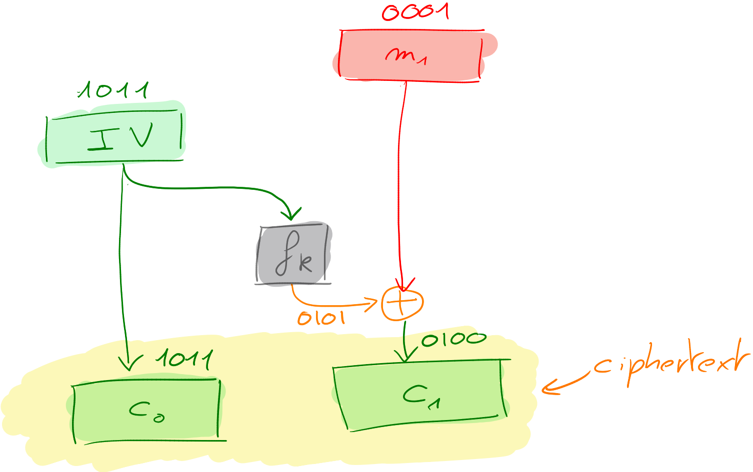

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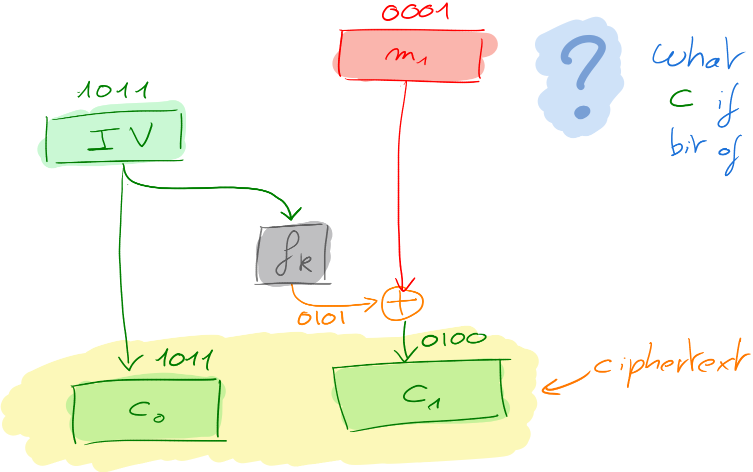
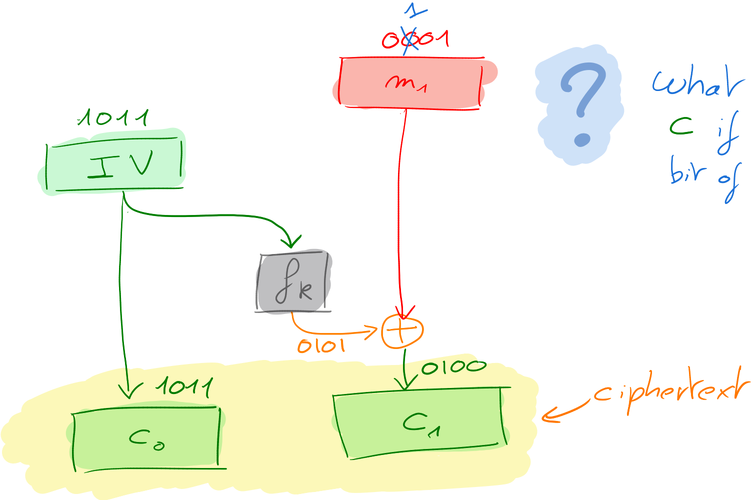


What happens to c if we flip one bit of m ?

Padding oracle attack



?

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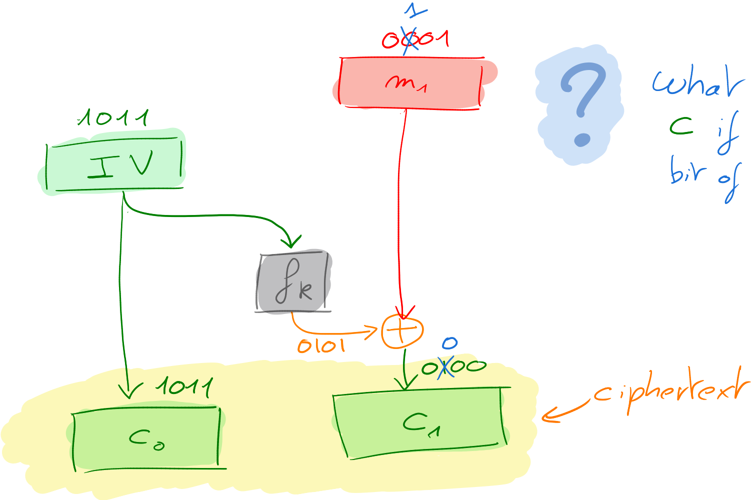

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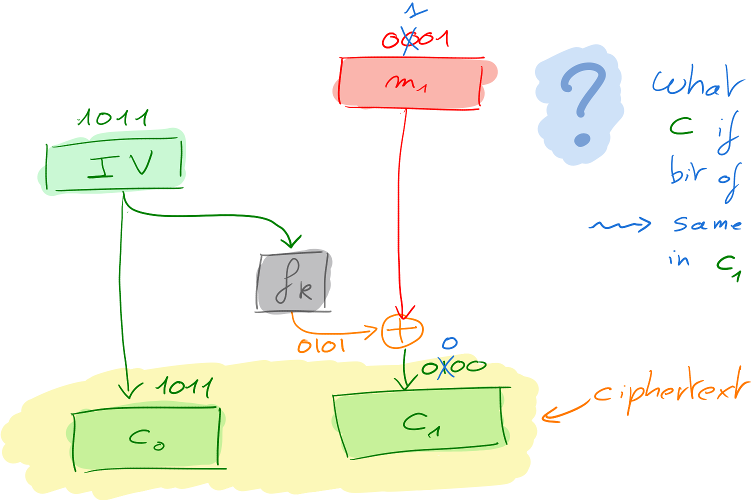
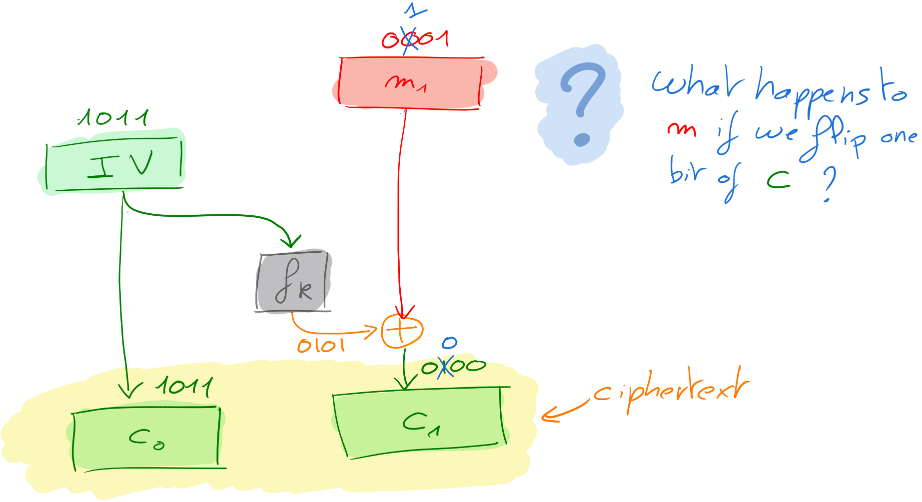


What happens to c if we flip one bit of m ?

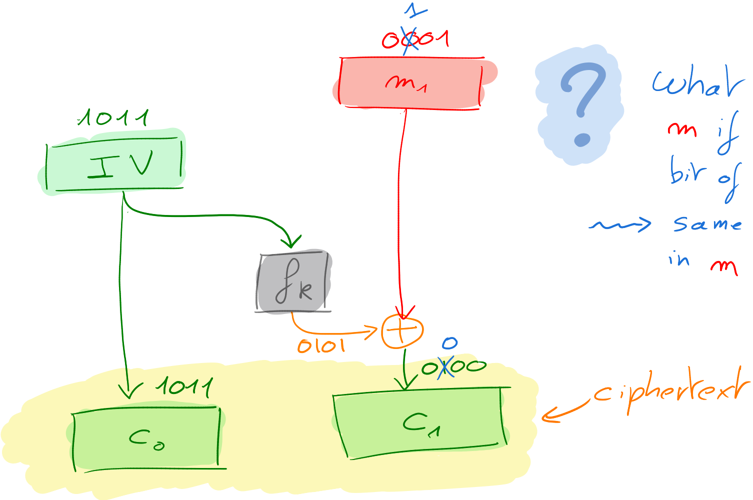
⇝ same bit is flipped in c_1 !!!

Padding oracle attack



What happens to m if we flip one bit of c ?

Padding oracle attack



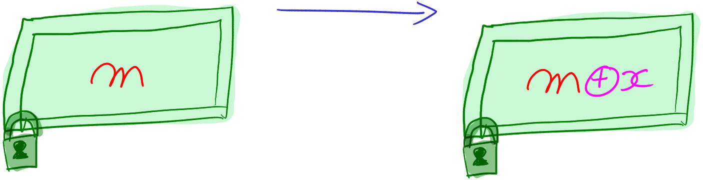
What happens to m if we flip one bit of c ?

⇝ same bit is flipped in m !!!

Padding oracle attack

$\forall x$, Attacker can turn:

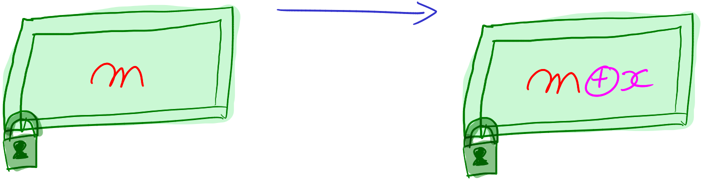
$\mathrm{Enc}_k(m)$ $\mathrm{Enc}_k(m \oplus x)$



Padding oracle attack

$\forall x$, Attacker can turn:

$$\mathrm{Enc}_k(m) \longrightarrow \overbrace{\mathrm{Enc}_k(m \oplus x)}^{= \mathrm{Enc}_k(m) \oplus (0-0 \| x)}$$



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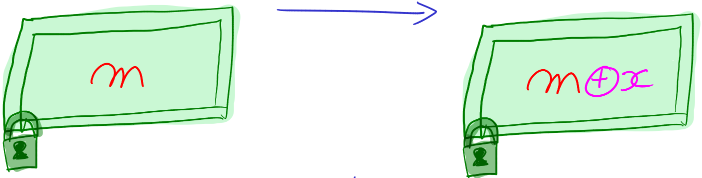

Idea: learn info on m by checking if $m \oplus x$ is well padded!

Padding oracle attack

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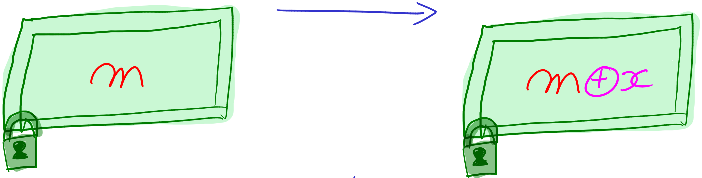

<u>Idea</u>: learn info on m by checking if $m \oplus x$ is well padded!

How?

Padding oracle attack

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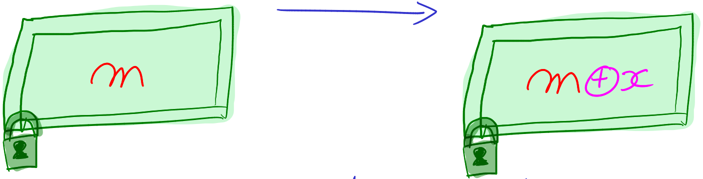

Idea: learn info on m by checking if $m \oplus x$ is well padded!

How? Given $\text{Enc}_k(m)$ send $\overbrace{\text{Enc}_k(m) \oplus (0-0\|x)}^{= \text{Enc}_k(m \oplus x)}$ to the padding oracle

Padding oracle attack

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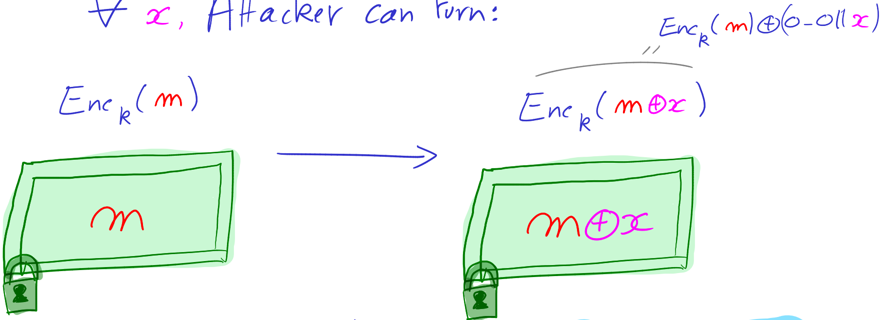

Idea: learn info on m by checking if $m \oplus x$ is well padded!

How? Given $Enc_k(m)$ send $\overbrace{Enc_k(m) \oplus (0-0 \| x)}^{= \, Enc_k(m \oplus x)}$ to the padding oracle

Padding oracle attack

By checking if $\underset{\text{unknown}}{m} \oplus \underset{\text{chosen by adversary}}{x}$ is well padded, how to learn m?

- Is m well padded?

Padding oracle attack

By checking if $m \oplus x$ is well padded, how to learn m?

- Is m well padded? ⟹ Yes (honest Alice)

E.g. $m = 65|66|67|0|0|0|0|\underbrace{5}_{1 \text{ byte}}$

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- Is $m \oplus 0|0|0|0|0|0|1|0$ well padded?

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- Is $m \oplus 0|0|0|0|0|0|1|0$ well padded?

  $= 65|66|67|0|0|0|1|5$

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By checking if $m \oplus x$ is well padded, how to learn m?

- Is m well padded? => Yes (honest Alice)

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  $= 65|66|67|0|0|0|1|5$ => X NO!

- Is $m \oplus 0|0|1|0|0|0|0|0$ well padded?

  $= 65|66|66|0|0|0|0|5$ => ✓ Yes!

Padding oracle attack

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- Is m well padded? $\Rightarrow$ Yes (honest Alice)

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- Is $m \oplus 0|0|0|0|0|0|1|0$ well padded?

  $= 65|66|67|0|0|0|1|5$ $\Rightarrow$ ✗ No!

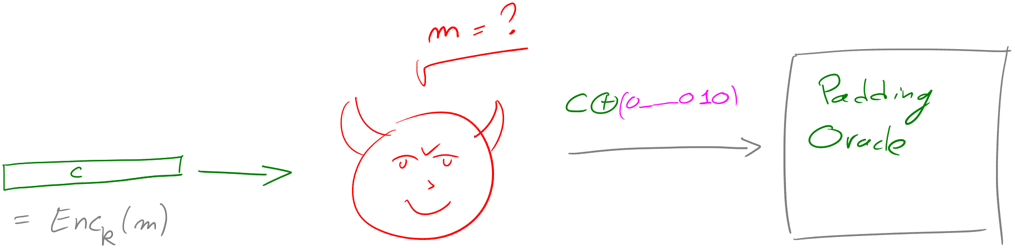
- Is $m \oplus 0|0|1|0|0|0|0|0$ well padded?

  $= 65|66|66|0|0|0|0|5$ $\Rightarrow$ ✓ Yes!

$\Rightarrow$ allows us to recover the length of m

Padding oracle attack

Padding oracle attack



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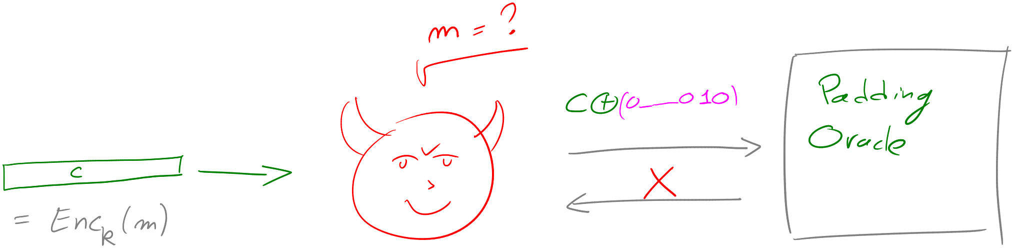

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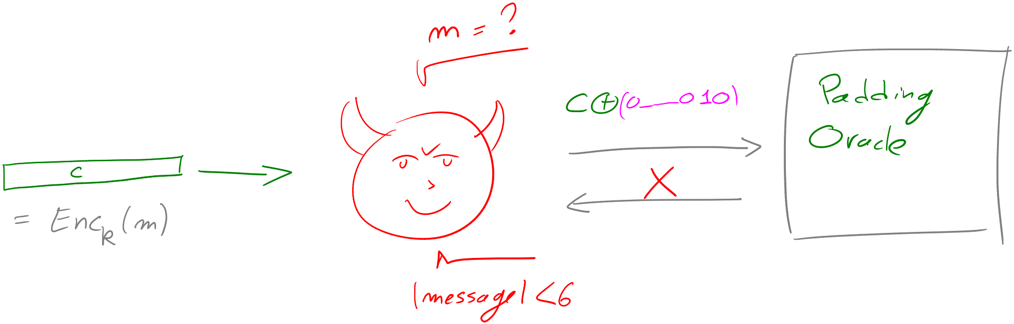

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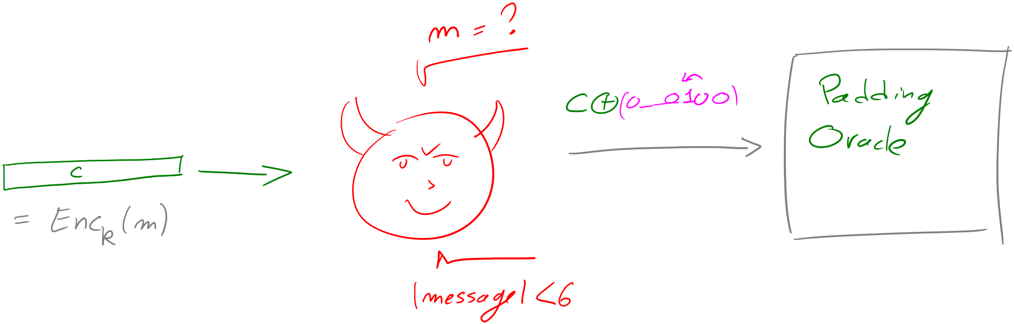

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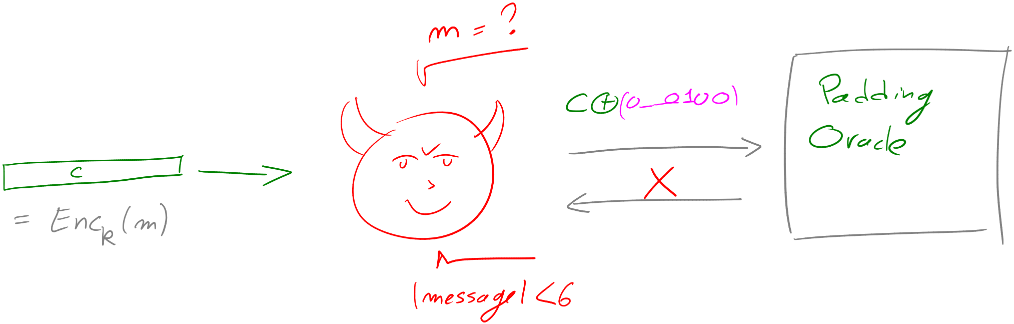

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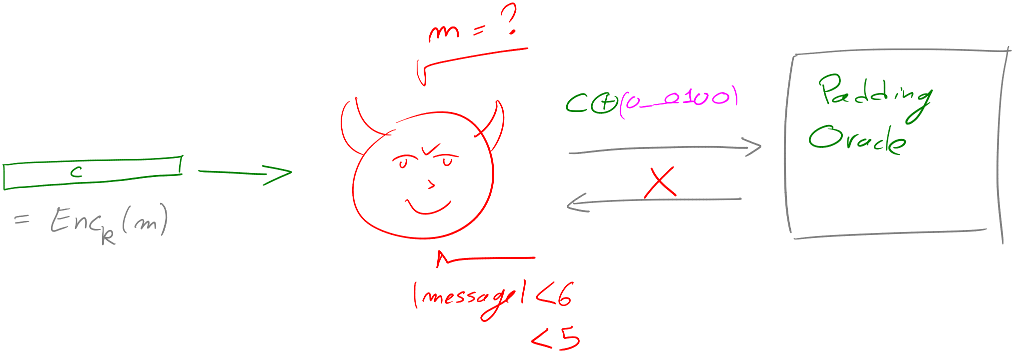

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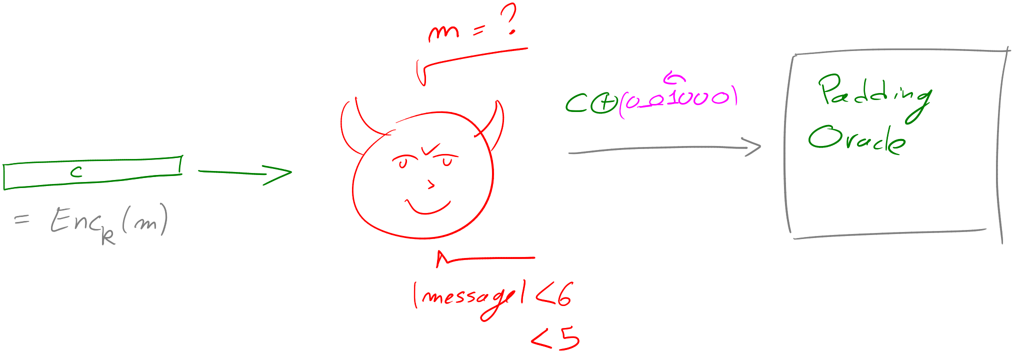

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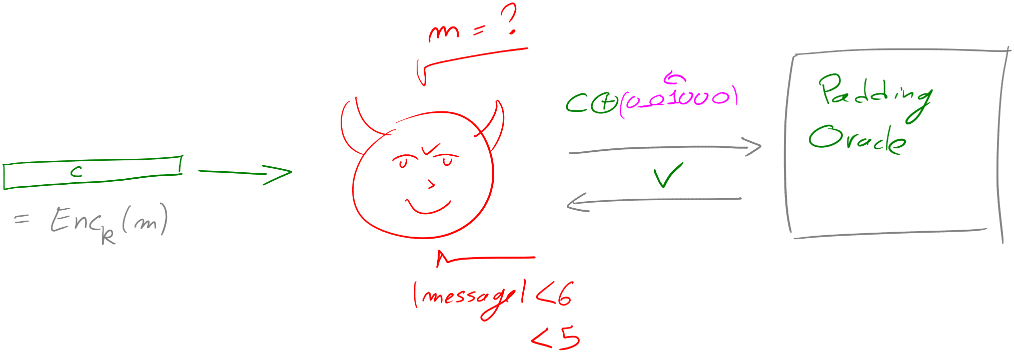

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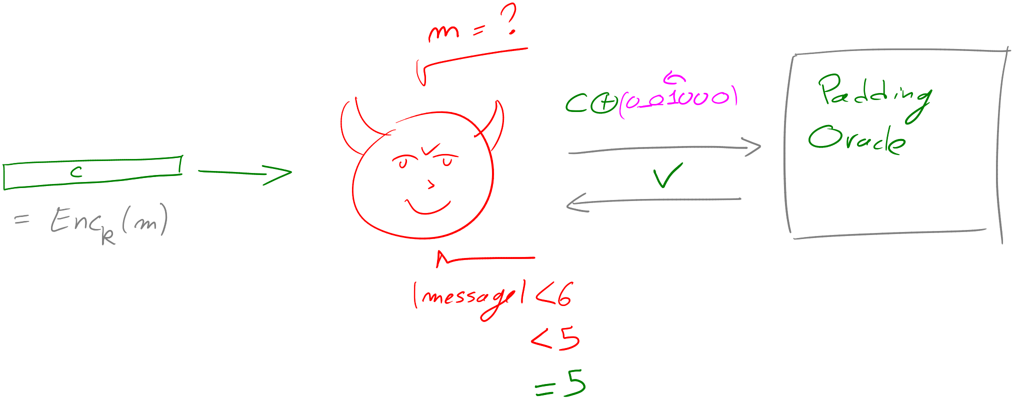

Padding oracle attack

How to learn the rest of m ?

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↳ attacker knows $|message|$ (e.g. 5)

so $m = ?|?|?|?|?|0|0|3$

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↳ $m \oplus 0|0|0|0|0|0|0|3 \oplus 4$

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$= ?|?|?|?|?|0|0|4$

↳ if valid padding, ? = 0, else ? ≠ 0 !

Padding oracle attack

How to learn the rest of m ?

↳ attacker knows |message| (e.g. 5)

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How to check if ? is $= \overset{x}{\cancel{0}}$?

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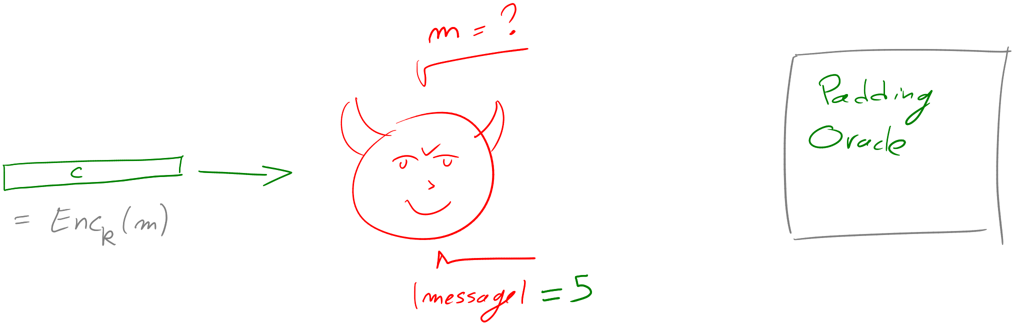
$= ?|?|?|?|? \oplus x|0|0|4$

↳ if valid padding, $? \oplus x = 0$, i.e. $? = x$

I just loop for all x !

!

Padding oracle attack



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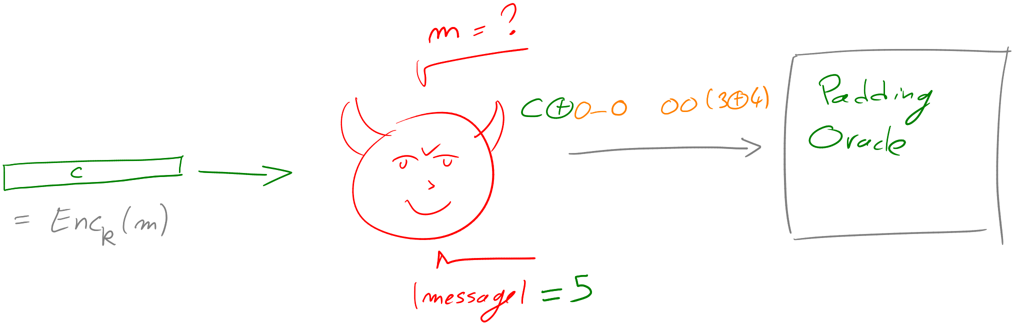

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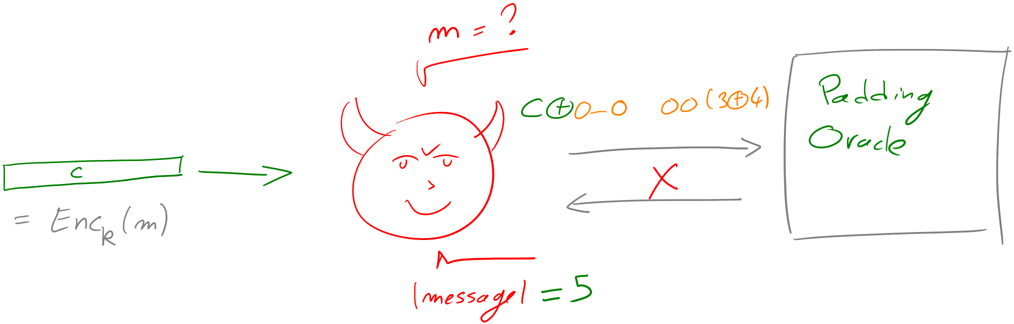

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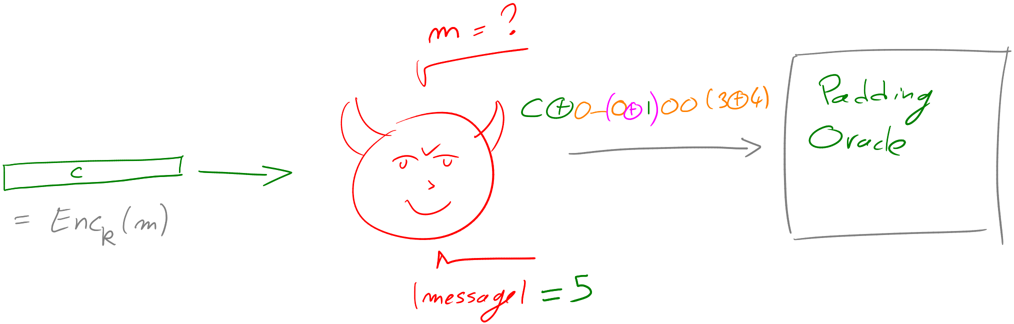

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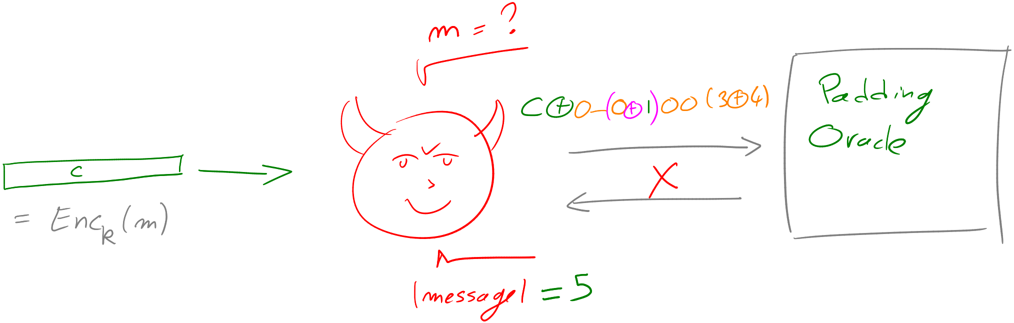

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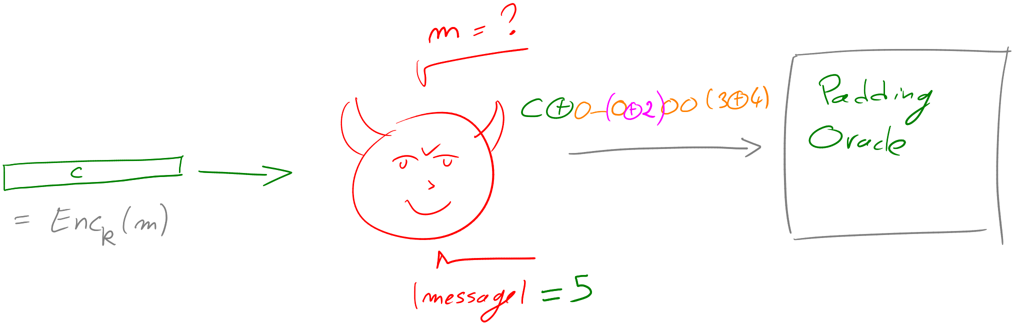

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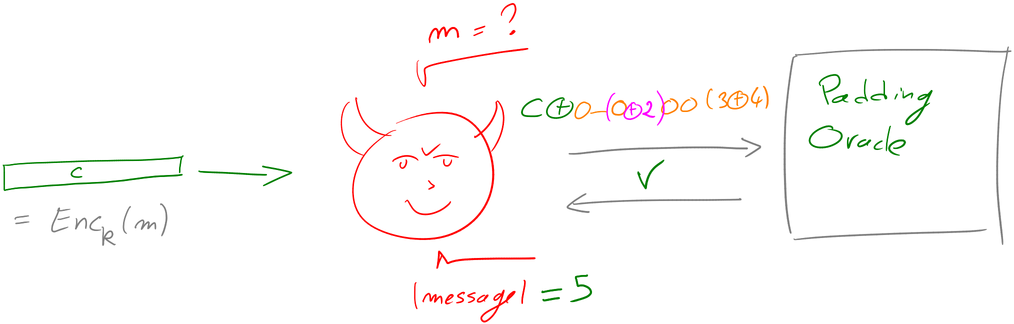

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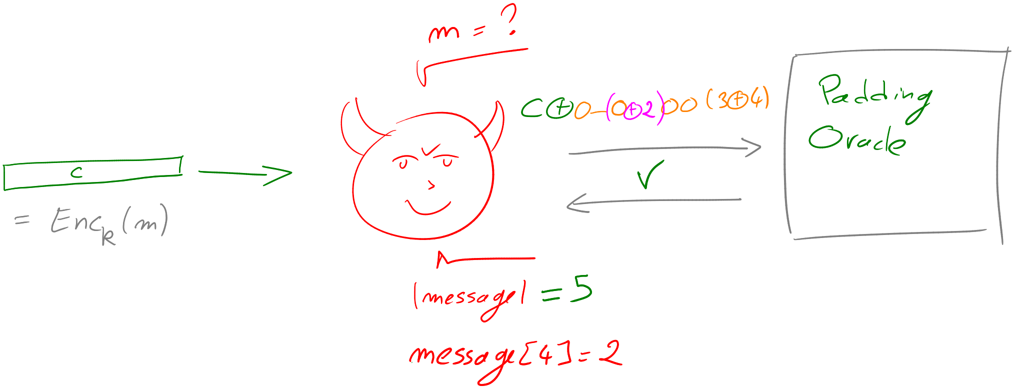
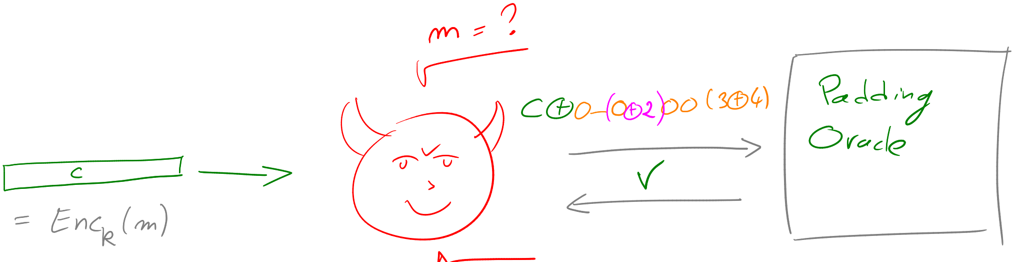


$|message| = 5$

$message[4] = 2$

Padding oracle attack

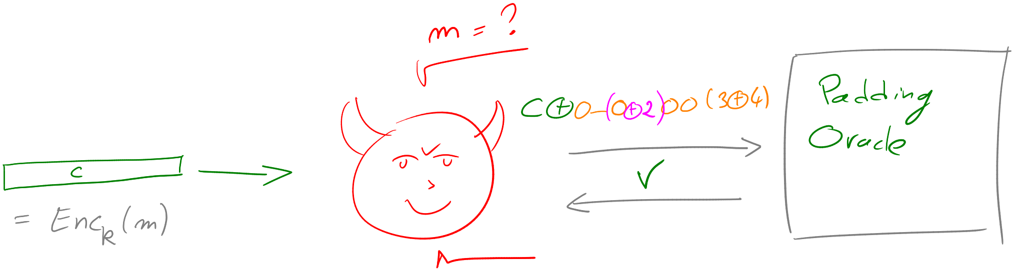


$|\text{message}| = 5$

$\text{message}[4] = 2$

and $c \oplus 0{-}(0 \oplus 2)00\,(3 \oplus 4) = \mathrm{Enc}(????\,000\,4)$

Padding oracle attack



$|message| = 5$

$message[4] = 2$

and $c \oplus 0-(0 \oplus 2)00(3 \oplus 4) = Enc(????0004)$

=> Repeat to extract all bits starting from ↰

Padding oracle attack

Limitation of IND-CPA security

Fundamental issue: not padding, but server behaves **differently based on the decrypted value**.

In practice, this is **extremely common** and hard to avoid (e.g. it takes maybe a bit longer to decrypt some messages, or does different operations based on the decrypted value…)

$\Rightarrow$ **We need a more resilient security definition**: allow attacker to decrypt arbitrary messages = IND-CCA!

IND-CCA security

IND-CCA

IND-CCA

Let Σ be an encryption scheme. We say that Σ has indistinguishable security against **chosen-ciphertext attacks (IND-CCA)** if:

```
$\mathcal{L}^{\Sigma}_{\text{cca-L}}$
$k \leftarrow \text{Gen}(1^\lambda)$
$\mathcal{S} := \emptyset$
EAVESDROP$(m_L, m_R \in \mathcal{M})$:
    if $|m_L| \neq |m_R|$ return err
    $c := \text{Enc}_k(m_L)$
    $\mathcal{S} := \mathcal{S} \cup \{c\}$
    return $c$
DECRYPT$(c \in \mathcal{C})$:
    if $c \in \mathcal{S}$ return err
    return Dec$(k, c)$
```

$$\approx$$

```
$\mathcal{L}^{\Sigma}_{\text{cca-R}}$
$k \leftarrow \text{Gen}(1^\lambda)$
$\mathcal{S} := \emptyset$
EAVESDROP$(m_L, m_R \in \mathcal{M})$:
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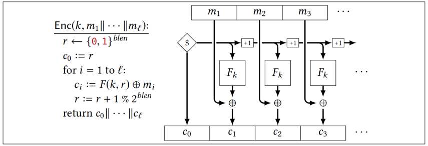
```

Malleability



```
Enc($k, m_1\|\cdots\|m_\ell$):
    $r \leftarrow \{0,1\}^{blen}$
    $c_0 := r$
    for $i = 1$ to $\ell$:
        $c_i := F(k,r) \oplus m_i$
        $r := r + 1 \% 2^{blen}$
    return $c_0\|\cdots\|c_\ell$
```

Can you find a CCA attack against, e.g., CTR mode?

- Ⓐ No
- Ⓑ Yes, withOui, avec

```
$\mathcal{A}$
$(c_0, c_1) \leftarrow$ EAVESDROP($0^{blen}, 1^{blen}$)
$m \leftarrow$ DECRYPT($(c_0, c_1 \oplus (10\ldots0))$)
return $m \overset{?}{=} 10\ldots0$
```

- Ⓒ Yes, with

```
$\mathcal{A}$
$(c_0, c_1) \leftarrow$ EAVESDROP($0^{blen}, 1^{blen}$)
$m \leftarrow$ DECRYPT($(c_0, c_1)$)
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```

Malleability

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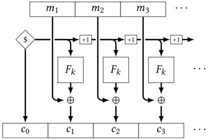

Can you find a CCA attack against, e.g., CTR mode?

- Ⓐ No ✗
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```
$\mathcal{A}$
$(c_0, c_1) \leftarrow$ EAVESDROP$(0^{\mathsf{blen}}, 1^{\mathsf{blen}})$
$m \leftarrow$ DECRYPT$((c_0, c_1 \oplus (10\ldots0)))$
return $m \overset{?}{=} 10\ldots0$
```

✓

- Ⓒ Yes, with

```
$\mathcal{A}$
$(c_0, c_1) \leftarrow$ EAVESDROP$(0^{\mathsf{blen}}, 1^{\mathsf{blen}})$
$m \leftarrow$ DECRYPT$((c_0, c_1))$
return $m = 0^{\mathsf{blen}}$
```

✗

Malleability

Fundamental reason: CTR is **malleable**, i.e. we can obtain $\mathsf{Enc}_k(x') = (c_0, x' \oplus F_k(c_0))$ from $\mathsf{Enc}_k(x) = (c_0, x \oplus F_k(c_0))$ (just add $x \oplus x'$ to the second element of the tuple).

Problem in real life: e.g. we can turn a “Yes” into a “No”.

How to prevent this? **Authentication!** (later course)

Conclusion

- OTP is statistically secure if **used once**
- A first notion of security against passive adversary is **IND-CPA**
- PRF $\Rightarrow$ IND-CPA secure schemes
- **Birthday paradox** = may need to double the size of key
- **Block-cipher modes** = encrypt efficiently arbitrarily long messages (padding sometimes necessary)
- CTR mode has good properties (but wait GCM)
- **AES** = standardized PRP (hence PRF) used in block-cipher modes
- **Malleable** encryption $\Rightarrow$ attacks against active adversaries (e.g. padding oracle/timing attacks)
- Spoiler: Authentication will help us!